IV.1) Rankine

In a paper read to the Philosophical Society of Glasgow, on the 5th January, 1853, Rankine introduced for the first time the term "potential energy" (1). Just six years after Helmholtz’s paper, Rankine changes the terminology of PCE as well as the conceptual meaning of the term. Rankine’s PCE2 asserts that "The quantity whose amount is ’conserved’ during all the mutual actions amongst a system of bodies is always the product of two factors; and when one of those factors is the magnitude of a tendency towards change of a particular kind, the other factor is the magnitude of the change throughout which that tendency is capable of continuing to act" (2). In Rankine there is no basic reference to Newtonian central forces as in Helmholtz, nor to an analytical complete differential as we will see in Clausius; i.e. these elements are not the main ones in his approach. Rankine’s conception is much more abstract than Helmholtz’s but not in the mathematical sense of Clausius. He is more inclined towards a "metaphysical" level, whose physical consequences were to be extremely important. In fact Rankine was the pioneer of the science of energetics, and we can find instances of his conception of "factorization of energy" in the work of Maxwell (1873) and Sommerfeld (1948) (3).

The word "energy" receives its first explicit mention just at the beginning of the 1853 paper:

"In this investigation the term energy is used to comprehend every affection of substances which constitutes or is commensurable with a power of producing change in opposition to resistance, and includes ordinary motion and mechanical power, known or unknown, which are convertible or commensurable with these. All conceivable forms of energy may be distinguished into two kinds, actual or sensible, and potential or latent." (4)

In 1859 Rankine noticed that the first use of the term energy "in this sense" is found in Young (Lectures on Natural Philosophy, Lecture VIII) and that after 1853 he found "that Carnot, in his essay on the principles of equilibrium and motion, proposed a pair of analogous terms in the case of mechanical energy, viz. ’virtual vis viva’ and ’actual vis viva’" (5).

In the paper of 1853 the definitions of actual and potential energy are as follows:

"Actual energy is a measurable, transferable, and transformable affection of a substance, the presence of which causes the substance to tend to change its state in one or more respects; by the occurrence of which changes, actual energy disappears, and is replaced by potential energy, which is measured by the amount of a change in the condition of a substance, and that of the tendency of force whereby that change is produced (or, what is the same thing, of the resistance overcome in producing it), taken jointly." (6)

The "metaphysical" approach is evident; actual and potential energy are defined in terms of tendency to change and changes of the states or conditions of substances - no reference is made to physical models nor to analytical conditions. The PCE is taken for granted:

"If the change whereby potential energy has been developed be exactly reversed, then as the potential energy disappears, the actual energy which had previously disappeared is already known, viz. that the sum of the actual and potential energies in the universe is unchangeable." (7)

Rankine’s PCE2 looks like a definition, i.e. the conservation seems to depend on the way in which actual and potential energy were defined. This criticism was in fact raised by John Herschel and below we will analyse Rankine’s answer in 1867. It is important to note that Rankine is extremely sophisticated from a methodological point of view, that his interest is mainly directed to a reformulation of the whole of physics in terms different from the mechanical ones, but more general than those linked with a specific realm of phenomena, e.g. electromagnetism, and that thus his papers on energy have to be read on three levels: physical, methodological and "metaphysical".

I shall not pursue the problem of the 1853 paper, i.e. the analytical formulation of a general law of transformation of energy between the actual and potential state, but I will analyse in some detail Rankine’s factorisation of actual and potential energy in the 1859 paper. According to Rankine "no law of ’conservation’ is applicable to the tendency of a body to change its place, nor to any mere tendency whatsoever". (8)

Moreover, Rankine asserts that his own PCE2 is "the only precisely-defined meaning which has ever been assigned to the word ’energy’ in writings on physical science." (9) In fact the old term ’conservation of force’ is ambiguous, since the word ’force’ is commonly used to denote a tendency and no conservation applies to "any tendency whatsoever". It is interesting to note how Rankine applies the factorisation to both forms of energy, potential and actual.

As to the actual energy, the argument is the following: the product of the mass into the half square of its velocity  is divided into two factors. The first represents "a force (in the sense of tendency)": mass x velocity/time of action, i.e. (mv/t), and the second represents the change, in this case a distance: half velocity x time of action, i.e. . The motion of a mass with given velocity is in fact actual energy, for it consists in a state of change going on (10).

Potential energy on the other hand consists in a tendency towards a change which is capable of continuing to act through the change. The magnitude of the tendency being variable, the mass value of that magnitude is the factor to be used in the multiplication: "or, what is the same thing in other words, that ... the integral of the tendency with respect to the change is the quantity in question." (11)

For example, in the case of a pair of gravitating bodies, if a is the sum of their radii and r the distance of their centres, their tendency to approach each other is:



and the potential energy:



that is "the product of the factor tendency Fmm'/ar1, the mean attraction through the distance, and the distance r1-a1 between the surfaces of the bodies. (12)

Work is thus understood in a completely different sense from the earlier interpretations: the main aspects of the definition of work are now its two factors, force and displacement; no reference is made to physical conditions imposed on the forces, nor to analytical conditions imposed on the work as differential, even though of course this last condition is to be found in the derivation of the general law of transformation of energy. In this case the condition would appear as a condition on the energies and not on the work:

"...we obtain the algebraical sum of the energies, actual and potential, received and developed by the substance during the changes dQ, dV; which is thus expressed:



This quantity must be the exact differential of a function of Q and V; for otherwise it would be possible, by varying the order of the increments dQ, dV, to change the sum of the energies of the universe" (13)

Here V is the magnitude of a measurable state of a substance; U the species of potential energy which is developed when the state V increases; P=dU/dV; Q the quantity which the substance possesses, of a species of actual energy whose presence produces a tendency of the state V to increase; LdQ is the actual energy which transforms itself into some unknown form, in consequence of the resistance which is offered to the increase of the total actual energy).

In fact, the reference to the concept "work" is basic. At the very end of the 1859 paper Rankine asserts that:

**"energy, or the quantity which remains constant in all physical actions amongst a system of bodies, is either the product of two factors - a tendency or effort to produce a change, and the change throughout which that effort is capable of continuing to act, or is equivalent to such a product." (14**)

Moreover, in a paper of 1855: "Outlines of the science of energetics", Rankine had already stated that "the term ’energy’ comprehends every state of a substance which constitutes a capacity for performing work. Quantities of energy are measured by the quantities of work which they constitute the means of performing." (15)

Work had earlier been defined as "a term comprehending all phenomena in which

physical change takes place." (16)

Energy, again, had been defined (17) also as the capacity to effect changes. This approach to PCE was to be influential, at least in part, on Maxwell. Being the more ’metaphysical’ among the different conceptions of energy we are examining, it is appropriate that we should give a short outline of Rankine’s conception of physical theories and of his metaphysical ideas on ’substances’ and ’accidents’, although a careful analysis of these topics is beyond the scope of the present work.

In the following discussion of Rankine’s methodological ideas, Einstein’s classification of theories will be used as reference. As is well known, (18) Einstein divides scientific theories into three groups; phenomenological theories, constructive theories and theories of principle. For instance, theory of heat is a phenomenological theory, kinetic theory of gases is constructive, and relativity theory is a theory of principle. Rankine starts his analysis by defining two steps in "the process of advancing our knowledge of the laws of physical phenomena". The first step "consists in observing the relations of phenomena ... and in expressing the relations so observed by propositions called formal laws". These relations in Rankine’s view are really supposed to be observed: the formal laws, although they are "in general numerous and complex" are "the immediate results of observation and experiment". They are, in fact, "deduced immediately from observation and experiment" (19)(20). Once the first stage is accomplished, i.e. the formal laws stated, the second stage has to be faced. It consists "in discovering the most simple system of principles, from which all the formal laws of the class of phenomena can be deduced as consequences". As opposed to the first stage in which the ’deduction’ of formal laws was immediate, the discovery of the principles is "to a certain extent, tentative - that is to say, involving the trial of conjectural principles". The axioms and definitions so discovered are accepted or rejected "according as their consequences are found to agree or disagree" with the previously mentioned formal laws (1). Again, contrary to the Einsteinian scheme (22), Rankine states a direct correspondence

between observational propositions and experiments, but in agreement with this scheme, he states a non-logical connection between observational propositions and the discovery of axioms. The system of principles obtained, together with its consequences methodically deduced, constitutes for Rankine the Physical Theory of a class of phenomena (23).

I next turn to another important point of great interest which concerns the definition of the first two methods of framing the theories: the abstractive and the hypothetical. As to the first, we again find a strict relation between sense perceptions and propositions: the peculiarity of this method, in fact, is the description of a class of phenomena through the properties common to all objects or phenomena of the class "as perceived by the senses, without introducing anything hypothetical". (This approach is close to the one defined by Einstein as phenomenological). The second method, on the other hand, defines object and phenomena "according to a conjectural conception of their nature, as being constituted, in a manner not apparent to the senses, by a modification of some other class of objects or phenomena whose laws are already known." (24)

Rankine’s propensity for a third method, "an extension of the abstractive method", is evident from his analysis of mechanics. In his view in fact, "the principles of the science of mechanics ... are altogether formed from the data of experience by the abstractive method". The objects of this science are the material bodies, i.e. bodies with the properties of occupying space and of resisting change of motion. The phenomena described by this science are defined by the two concepts of motion and force. Force is considered a tendency towards or against motion. "The law of the relations between motion and force are the consequences of certain axioms, being the most simple and general expressions for all that has been ascertained by experience respecting those relations." (25). In this statement Rankine seems to incline towards the principles of simplicity and generality as links between observable propositions and axioms. But the stress is on the ’abstraction’ from experience.

According to Rankine, the hypothetical method found expression after the abstractive.

In fact, for Rankine, it was just the success of the abstractive method that led to mechanics. At the time, it was the only complete physical theory which allowed the extension of (mechanical) hypotheses into other fields of inquiry, i.e. it led to the hypothetical method. In fact hypothetical motions and forces, such as the molecular ones, were sometimes attributed to hypothetical bodies, such as the luminiferous ether, sometimes to hypothetical parts, such as atoms and atomic nuclei (26). In my view, Rankine’s hypothetical method is close to the one defined by Einstein as constructive.

Of course this aspect of Rankine’s work can be criticised in view of the fact that atomistic theories were much older than he seems to think, and that Huygens, Descartes and Newton used hypothetical methods, but it is instructive nonetheless to follow his discussion. Rankine outlines advantages and disadvantages of the hypothetical theories, but is mainly interested in sketching an extension of the abstractive method. The extension is described in this way:

"Instead of supposing the various classes of physical phenomena to be constituted, in an occult way, of modifications of motion and force, let us distinguish the properties which those classes possess in common with each other, and so define extensive classes denoted by suitable terms. For axioms, to express the law of those more extensive classes of phenomena, let us frame propositions comprehending as particular cases the laws of the particular classes of phenomena comprehended under the more extensive classes. So shall we arrive at a body of principles, applicable to physical phenomena in general, and which, being framed by induction from facts alone, will be free from the uncertainty which must always attach, even to those mechanical hypotheses whose consequences are most fully confirmed by experiment." (27)

In this "extension of the abstractive method" we can recognise Einstein’s theories of principle, even if, in Einstein’s view, the principles are not inductively derived from facts. Rankine’s approach deserves two comments: a) it looks like an ad hoc foundation of "the science of energetics"; in fact energy is a property common to various classes of physical phenomena which can be used to define more extensive classes "denoted by suitable terms"; b) the idea of properties directly perceived by the senses is closely connected with a specific "observable" meaning of energy. In fact Rankine, in the following remarks, introduces elements that lead one to think that the properties he is speaking of are not so directly perceivable by the senses.

"This extension of the abstractive process is not proposed in order to supersede the hypothetical method of theorising; for in almost every branch of molecular physics it may be held, that a hypothetical theory is necessary, as a preliminary step, to reduce the expression of the phenomena to simplicity and order, before it is possible to make any progress in framing an abstractive theory." (28)

In my view the only possibility of attributing to energy an "observable" meaning lies for Rankine in the full acceptance of the previous results of the hypothetical mechanical method. That is, a theoretical term (energy) becomes observable if considered from a more abstractive point of view in the light of mechanical models. Rankine’s classification of theories is thus also a classification towards a higher level of abstraction: from the abstract to the hypothetical to the generalised abstract. The three are in immediate correspondence with the Einsteinian classification: phenomenological, constructive, principles.

In addition, they correspond to the three components of the neo-Kantian scheme of Buchdahl: physical, metaphysical, systematical (29). In Buchdahl’s view these three elements are at the same time components of a scientific theory and points of view of epistemological analysis. In fact also in Rankine’s view we saw that the three methods are interconnected and the preference accorded to one or the other depends on physical and philosophical aspects. As to physical aspects, emphasis on the regulative and systematical component is useful only in a mature theory. It is not a coincidence that Rankine classifies the "generalised abstractive method" as the third, nor that Einstein showed his interest in the theories of principle after Lorentz’s "constructive" synthesis of electromagnetic phenomena. Rankine explicitly utilised a PCE already in existence. He always refers to PCE as something given. He is mainly interested in a reformulation of PCE1 to provide a better account for "transformations" of energy and in reframing physical theories under a PCE framework. Thus he is using PCE as a systematical or regulative component, in Buchdahl’s language. In such a shift, of course, PCE and the concept of energy undergo a shift in meaning; they become at the same time less of an empirical theory (Poincarè analysed the impossibility of refuting PCE on empirical grounds, once it had been established (30)), but more evident a priori, also at an "observable" level. In fact they are now considered immediately recognisable "common" properties of the bodies. The regulative use of the principle has achieved its results.

In my view the previous comments are corroborated by Rankine’s subsequent statements:

"Energy, or the capacity to effect changes, is the common characteristic of the various states of matter to which the several branches of physics relate." (31)

Only eight years after Helmholtz’s 1847 paper, a very important shift has occurred in relation to the meaning of energy. The systemic and regulative approach is expressed thus:

"if, then, there be general laws respecting energy, such laws must be applicable, mutatis mutandis, to every branch of physics, and must express a body of principles as to physical phenomena in general." (32)

On the basis of these principles, Rankine was to outline the "science of energetics". A peculiarity of his approach is the reference to classes of objects and phenomena and not to specific objects or phenomena. Thus he defined general terms such as substance; property; mass; active, passive and radical accident; effort. (33)

In the definitions of work and energy we find the conception of factorisation: work is defined as the variation of an accident by an effort, and energy is a capacity for performing work. The difference between work and potential energy is that the former has been effected while the latter is capable of being effected.

The already mentioned conceptual equivalence between work and energy, the root of Rankine’s factorisation of energy, finds explicit expression in the first axiom of the new science: "All kinds of work and energy are homogeneous". (34)

As a last comment by Rankin on his methodological approach, I will quote from his remarks on this axiom, which give evidence for the difference between the hypothetical and

generalised abstractive method. In fact, in Rankine’s view, the meaning of the axiom is immediately recognisable as observable and factual:

"This axiom means, that any kind of energy may be made the means of performing any kind of work. It is a fact arrived at by induction from experiment and observation, and its establishment is more especially due to the experiments of Mr. Joule." (35)

This "fact" could be interpreted by means of the hypothetical method, e.g; assuming molecular motions:

"This axiom leads, in many respects, to the same consequences with the hypothesis that all those kinds of energy which are not sensibly the results of motion and motive force are results of occult modifications of motion and motive force." (36)

But for Rankine there is no need to assume this "unobservable" hypothesis: the axiom is at the same time general and close to the facts:

"But the axiom differs from the hypothesis in this, that the axiom is simply the generalised allegation of the facts proved by experience, while the hypothesis involves conjectures as to objects and phenomena which never can be subjected to observation." (37)

The previous quotation in my view explains the difference between Rankine’s and Einstein’s classification of theories. For Rankine, a "generalised abstractive method" is possible, that is, the generalisation does not imply a renunciation of the "factual" content of the laws. At the same time, for Rankine, the results of the hypothetical method are necessary: the factual generalisation is made possible by the results of the hypothetical method. In Einstein, instead, the phenomenological theories and the theories of principles are sharply divided; no inductive tools can produce the axioms.

The matter is somewhat clarified by Rankine’s answer, in 1867, to a criticism of Sir John Herschel. For Herschel, the phrase Potential Energy is "unfortunate, inasmuch as it goes to substitute a truism for the announcement of a great dynamical fact" (Familiar Lectures, p. 469) (38).

In his answer Rankine does not refute the objection that his definition seems a truism: he contrasts the negative judgment with the need for a "truistic definition". In this way he outlines a very interesting relation between definitions and facts:

"A definition cannot be true or false, for it makes no assertion, but says, ’Let such a word or phrase be used in such a sense’; but it may be real or fantastic, according as the description contained in it corresponds, or not, to real objects and phenomena, and when, by the aid of experiment and observation, a set of definitions have been framed which possess reality, precision, and completeness, the investing of a physical fact with the appearance of a truism is often an unavoidable consequence of the use of the term so defined." (39)

and:

"but when it is proved by experiment and observation that there are relations amongst real bodies agreeing with the definitions of ’actual energy’ and ’potential energy’, that which otherwise would be a truism becomes a fact." (40)

Rankine quotes as previous examples of his method "the definitions of the measurement of time, force, and mass, which reduce the laws of motion to the form of truism" and concludes:

"In conclusion, it appears to me that the making of a physical law wears the appearance of a truism, so far from being a ground of objection to the definition of a physical term, is rather a proof that such definition has been framed in strict accordance with reality." (41)