36.] ELECTRICITY **AS A** QUANTITY. 39

the result of the action between the parts of an intervening medium, it is conceivable that in all cases of the increase or diminution of the energy within a closed surface we may be able, when the nature of this action of the parts of the medium is clearly understood, to trace the passage of the energy in or out through that surface.

There is, however, another reason which warrants us in asserting that electricity, as a physical quantity, synonymous with the total electrification of a body, is not, like heat, a form of energy. An electrified system has a certain amount energy, and this energy can be calculated by multiplying the quantity of electricity in each of its parts by another physical quantity, called the Potential of that part, and taking half the sum of the products. The quantities 'Electricity' and 'Potential,' when multiplied together, produce the quantity 'Energy.' It is impossible, therefore, that electricity and energy should be quantities of the same category, for electricity is only one of the factors of energy, the other factor being 'Potential.'\*

**Energy, which is the product of these factors, may also be considered as the product of several other pairs of factors,**

**such as**

**• Force x A distance through which the force is to act.**

**• Mass x Gravitation acting through a certain height.**

**• Mass x Half the square of its velocity.**

**• Pressure x A volume of fluid introduced into a vessel at that pressure.**

• Chemical Affinity x A chemical change. measured by the number of electro-chemical equivalents which enter into combination.

If we ever should obtain distinct mechanical ideas of the nature of electric potential, we may combine these with the idea of energy to determine the physical category in which 'Electricity' is to be placed.

36.] In most theories on the subject, Electricity is treated as a substance, but inasmuch as there are two kinds of electrification which, being combined, annul each other, and since we cannot conceive of two substances annulling each other, a distinction has been drawn between Free Electricity and Combined Electricity.

\* {It **is shown afterwards** that 1 Potential' **is not of zero dimensions.}**

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every substance which we can act upon with the means at our disposal.

If an electrified body be placed at any part of the electric field it will-, In general, produce a sensible disturbance in the electrification of the other bodies.

But if the body is very small, and its charge also very small, the electrification of the other bodies will not be sensibly disturbed, and we may consider the position of the body as determined b its centre of mass. The force acting on the body will then be proportional to its charge, and will be reversed when the charge is reversed.

Let e be the charge of the body, and F the force actino, on the body in a certain direction, then when e is very small F is propor-

tional to e, or *F= Be,*

where B depends on the distribution of electricity on the other bodies in the field. If the charge e could be made equal to unity without disturbing the electrification of other bodies we should have F=R.

We shall call B the Resultant Electromotive Intensity at the given point of the 'field. When we wish to express the fact that this quantity is a vector we shall denote it by the German letter Q.

*Total Electroniotive Force* and Potential.

45.] If the small body carrying the small charge e be moved from one given point, A, to another B, along a given path, it will experience at each point of its course a force Be, where B varies from point to point of the course. Let the whole work done on the body by the electrical force be Ee, then E is called the Total Electromotive Force along the path *AB.* If the path forms a complete circuit, and if the total electromotive force round the circuit does not vanish, the electricity cannot be in equilibrium but a current will be produced. Hence in Electrostatics the total electromotive force round any closed circuit must be zero, so that if A and B are two points on the circuit, the total electromotive force from A to B is the same along either of the two paths into which the circuit is broken, and since either of these can be altered independently of the other, the total electromotive force from A to *B* is the same for all paths from A to B.

*If B* is tal,-en as a point of reference for all other points, then the total electromotive force from A to B is called the Potential of *A.*

46.] ELECTRIC POTENTIAL. 49

It depends only on the position of *A.* In mathematical investi-

gations, B is generally taken at an infinite distance from the

electrified bodies.

A body charged positively tends to move from places of greater positive potential to places of smaller positive, or of negative, potential, and a body charged negatively tends to move in the opposite direction.

In a conductor the electrification is free to move relatively to the conductor. If therefore two parts of a conductor have different potentials, positive electricity will move from the part having greater potential to the part having less potential as long as that difference continues. A conductor therefore cannot be in electrical equilibrium unless every point in it has the same potential. This potential is called the Potential of the Conductor.

Equipotential *Surfaces.*

46.] If a surface described or supposed to be described in the electric field is such that the electric potential is the same at every point of the surface it is called an Equipotential surface.

An electrified particle constrained to rest upon such a surface will have no tendency to move from one part of the surface to another, because the potential is the same at every point. An equipotential surface is therefore a surface of equilibrium or a level surface.

The resultant force at any point of the surface is in the direction of the normal to the surface, and the magnitude of the force is such that the work done on an electrical uiait in passing from the surface V to the surface -F is V- *V.*

No two equipotential surfaces bavidv different potentials can meet one another, because the same poi@'t cannot have more than one potential, but one equipotential surface may meet itself, and this takes place at all points and along all lines of equilibrium.

The surface of a conductor in electrical equilibrium is necessarily an equipotential surface. If the electrification of the conductor is positive over the whole surface, then the potential will diminish as we move away from the surface on every side, and the conductor will be surrounded by a series of surfaces of lower potential.

But if (owing to the action of external electrified bodies) some

**50** ELECTROSTATIC PHENOMENA. [46.

regions of the conductor are charged positively and others negatively, the complete equipotential surface will consist of the surface of the conductor itself.- ogether with a system of other surfaces, meeting the surface-of the conductor in the lines which divide the positive from the @egative regions \*. These lines will be lines of equilibrium, and an electrified particle placed on one of these lines will experience no force in any direction.

When the surface of a conductor is charged positively in some parts and negatively in others, there must be some other electrifled body in the field besides itself For if we allow a positively electrified particle, starting from a positively charged part of the surface, to move always in the direction of the resultant force upon it, the potential at the particle will continually diminish till the particle reaches either a negatively charged surface at a potential less than that of the first conductor, or moves off to an infinite distance. Since the potential at an infinite distance is zero, the

latter case can only occur when the potential of the conductor is positive.

In the same way a negatively electrified particle, moving off from a negatively charged part of the surface, must either reach a positively charged surftce, or pass off to infinity, and the latter case can only happen when the potential of the conductor is negative.

Therefore, if both positive and negative charges exist on a conductor, there must be some other body in the field whose potential has the same sign as that of the conductor but a greater numerical value, and if a conductor of any form is alone in the field the charge of every part is of the same si,)n as the potential of the conductor.

The interior surface of a hollow conducting vessel containing no charged bodies is entirely free from charge. For if any part of the surface were charged positively, a positively electrified particle moving in the direction of the force upon it, must reach a negatively charged surface at a lower potential. ]3ut the whole interior surface has the same potential. Hence it can have no chai@(-,e

@See Arts. RO, 114.

t To mal e the proof rigid it is necessary to state that by Art. 80 the force cannot

Vllnis where'tlie surftce **ii** cli'trge(l, an(I that by Art. 112 the potential cannot have t

maximum or minimum value at a point where there is no electrification.'f

9.1 ELECTRIC TENSION.

A conductor placed inside the vessel and communicating with it, may be considered as bounded by the interior surface. Hence such a conductor has no charge.

*Lines of Foi,ce.*

47.] The line described by a point moving always in the direction of the resultant intensity is called a Line of Force. It cuts the equipotential surfaces at right angles. The properties of lines of force will be more fully explained afterwards, because Faraday has expressed many of the laws of electrical action in terms of his conception of lines of force drawn in the electric field, and indicating both the direction and the intensity at every point.

*Electric Tension.*

48.] Since the surface of a conductor is an equipotential surface,

the resultant intensity is normal to the surface, and it will be shewn in Art. 80 that it is proportional to the superficial density of the electrification. Hence the electricity on any **siii,%11** area of the surface will be acted on 'by a force tending.fron?, the conductor and proportional to the product of the resultant intensity and the density, that is, proportional to the square of the resultant intensity.

This force, which acts outwards as a tension on every part of the conductor, will be called electric Tension. It is measured like ordinary mecha:n ical tension, by the force exerted on unit of area. The word Tension has been used by electricians in several vague senses, and it has been attempted to adopt it in mathematical ;'language as a synonym for Potential; but on examinid the cases 9

in which tile word has been used, 1 think it will be more consistent with usage and with mechanical analogy to understand by tension a pulling force of so many pounds weight per square inch exerted on the surface of a conductor or elsewhere. We shall find that the conception of Faraday, that this electric tension

exists not only at the electrified surface but all L@.lono-, the lines of @ @'t

force, leads to a theory of electric action as a phenomenon of stress in a medium.

*F,Ieet.poi7iotive Fog,ce.*

49.] When two conductors at different potentials are connected by a thin conducting wire, the tendency of electricity to flow

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alone, the wire is measured by the difference of the potentials of

the two bodies. The difference of potentials between two con-

diietors or two points is therefore called the Electromotive force

between them.

Electromotive force cai:t'not **in** all cases be expressed in the form of a difference of potentials. These cases, however, are not treated of in Electrostatics. We shall consider them when we **come** to heterogeneous circuits, chemical actions, motions of iiia(ynets, inequalities of temperature, &c.

*Caliacity of a Conductor.*

50.] If one conductor is insulated while all the surroundinconductors are kept at the zero potential by being put in communication with the earth, and if the conductor, when charged with a quantity E of electricity, has a potential -F, the ratio of E to V is called the Capacity of the conductor. If the conductor is completely enclosed within a conducting vessel without touching it, then the charge on the inner conductor will be equal and opposite to the charge on the inner surface of the outer conductor, and will be equal to the capacity of'the inner conductor multiplied by the difference of the potentials of the two conductors.

*Electric* Accumulators.

A system consisting of two conductors whose opposed surfaces are separated from each other by a thin stratum of an insulating medium is called an electric Accumulator. The two conductors are called the Electrodes and the insulating medium is called the Dielectric. The capacity of the accumulator is directly proportional to thearea of the opposed surfaces and inverselyproportional to the thickness of the stratum between theni. A Leyden jar is an accumulator in which glass is the insulating medium. Accuinulators are sometimes called Condensers, but 1 prefer to restrict the term condenser to an instrument which is used not to hold electricity but to increase its superficial density.

PROPERTIES OF BODIES **IN** RELATION TO STATICAL ELECTRICITY.

*Resi,,ta?tce to the Pasage of Electricity thi,ouglt a Body.* 51.] When a charge of electricity is communicated to any part of a mass of metal the electricity is rapidly transferred from places of high to places of' low potential till the potential of the wh

72.] POTENTIAL. 79

conductor. Hence R *=* 0 throughout the whole space occupied by the conductor. From this it follows that

*dY* *dY* *dY*

- 0, *T =* 0, 0

*dx =* y

and therefore for every point of the conductor

where C is a constant quantity.

Since the potential at all points within the substance of the conductor is C, the quantity C is called the Potential of the conductor. C may be defined as the work which must be done by external agency in order to bring a unit of electricity from an infinite distance to the conductor, the distribution of electricity being supposed not to be disturbed by the presence of the unit \*.

It will be shewn at Art. 246 that in general when two bodies of different kinds are in contact, an electromotive force acts from one to the other throuo,,h the surface of contact, so that when they are in equilibrium the potential of the latter is higher than that of the former. For the present, therefore, we shall suppose all our conductors made of the same metal, and at the same

temperature.

If the potentials of the conductors A and B be T'A and Vi@ respectively, then the electromotive force along a wire joining A and B will be

in the direction AB, that is, positive electricity will tend to pass from the conductor of higher potential to the other.

Potential, in electrical science, has the same relation to Electricity thtt Pressure, in Hydrostatics, has to Fluid, or that Temperature, in Thermodynamics, has to He-,Lt. Electricity, Fluids, and Heat all tend to pass from one place to another, if the Potential, Pressure, or Temperature is greater in the first place than in the second. A fluid is certainly a substance, heat is as certainly not a substance, so that though we may find assistance from analogies of this kind in formin(y clear ideas of formal relations of electrical quantities, we ffiust be careful not to let the one or the other analogy sul@iest to us that electricity is either a substance like water, or a state of agitation like heat.

\* {If there is any discontinuity in the potential as we pass from the dielectric to the conductor it is necessary to state **whether** the electrified point is broti,lit inside tile conductor or merely to the surface.

98 ELECTROSTATICS. [82.

The interior surface of a closed conducting vessel is entirely free from charge, and the potential at every point within it is the same as that of the con@ae-,tor, provided there is no insulated and charged body within tile vessel.

For since a line of force must begin at a positively charged surface and end at a negatively charged surface, and since no charged body is within the vessel, a line of force, if it exists within the vessel, must begin and end on the interior surface of the vessel itself.

But the potential must be higher at the beginning of a **line** of force than at the end of the line, whereas we have proved that the potential at all points of a conductor is the same.

Hence no line of force can exist in the space within a hollow conducting vessel, provided no charo,,ed body be placed inside it.

If a conductor within a closed hollow conducting vessel is placed in communication with the vessel, its potential becomes the same as that of the vessel, and its surface becomes continuous with the inner surface of the vessel. The conductor is therefore free from cliar(ve.

If we suppose any charged surface divided into elementary portions such that the charge of each element is unity, and if solenoids having these elements for their bases are drawn through the field of force, then the surface-integral for any other surface will be represented by the number of solenoids which it cuts. It is in this sense that Faraday uses his conception of lines of force to indicate not only the direction but the amount of the force at any place in the field.

We have used the phrase Lines of Force because it has been used by Faraday and others. In strictness, however, these lines should be called Lines of Electric Induction.

In the ordinary cases the lines of induction indicate the direetion and magnitude of the resultant electromotive intensity at every point, because the intensit and the induction are in the same direction and in a constant ratio. There are other cases, however, in which it is important to remember that these lines indicate primarily the induction, and that the intensity is directl indicated by the equipotential surfaces, being normal to these surfaces and inversely proportional to the distances of consecutive surfaces.

164 MECI[ANICAL ACTION. [log.

radius is very great and equal to a cannot exceed **e'-/2a2,** and

when a is increased without limit, the surface-integral must be. come ultimately zero.

But this surface-int76gral is equal to the voliime-integral (26), and the value of !is volume-integral is the same whatever be the size of the space enclosed within 8, provided 8 encloses every point at which Vlqj differs from zero. Hence, since the inteoral is zero when a is infinite, it must also be zero when the limits of integration are defined by any surface which includes every point at which V2,P differs from zero.

109.] The distribution of stress considered in this chapter is precisely that to which Faraday was led in his investigation

of induction through dielectrics. He sums up in the following

words:-

' (1297) The direct inductive force, which may be conceived to be exerted in lines between the two limiting and charged conducting surfaces, is accompanied by a lateral or transverse force equivalent to a dilatation or repulsion of these representative lines (1224); or the attractive force which exists amongst the particles of the dielectric in the direction of the induction is accompanied by a repulsive or a diverting force in the transverse direction.

1(1298) Induction appears to consist in a certain polarized state of the particles, into which they are thrown by the electrified body sustaining the action, the particles assumino, positive and negative points or parts, which are symmetrically arianaed with respect to each other and the inducting surfaces or particles. The state must be a forced one, for it is originated and sustained only by force, and sinks to the normal or quiescent state when that 1'oree is removed. It can be continued only in insulators 'by the same portion of electricity, because they only can retain this state of the particles.'

This is an exact account of the conclusions to which we have been conducted by our mathematical investigation. At every point of the medium there is a state of stress such that there is tension along the lines of force and pressure in all directions at right angles to these lines, the numerical magnitude of the pressure beino. equal to that of the tension, and both varying as the square of the resultant force at the point.

The expression 1 electric tension' has been used in various

PARADAYS TIIFORY. 165

senses by different writers. I shall always use it to denote the tension along the lines of force, which, as we have seen, varies from point to point, and is always proportional to the square of

the resultant force at the point.

1 10.] The hypothesis that a state of stress of this kind exists in a fluid dielectric, such as air or turpentine, may at first sight appear at variance with the established principle that at any point in a fluid the pressures in all directions are equal. But in the deduction of this principle from a consideration of the mobility and equilibrium of the parts of the fluid it is taken for granted that no action such as that which we here suppose to take place along the lines of force exists in the fluid. The state of stress which we have been studying is perfectly consistent with the mobility and equilibrium of the fluid, for we have seen that, if any portion of the fluid is devoid of electric charge, it experiences no resultant force from the stresses on its surface, however intense these may be. It is only when a portion of the fluid becomes charged that its equilibrium is disturbed by the stresses on its surface, and we know that in this case it actually tends to move. Hence the supposed state of stress is not inconsistent with the equilibrium of a fluid dielectric.

The quantity W, which was investigated in Chapter IV, Art. 99a, may be interpreted as the energy in the medium due to the distribution of stress. It appears from the theorems of that chapter that the distribution of stress which satisfies the conditions there given also makes ].V an absolute minimum. Now when the energy is a minimum for any configuration, that configuration is one of equilibrium, and the equilibrium is stable. Hence the dielectric, when subjected to the inductive action of electrffied bodies, will of itself take up a state of stress distributed

in the way we have described

'@@lt must be carefully borne in mind that we have made only one step in the theory of the action of the medium. We have supposed it to be in a state of stress, but we have not in any way accounted for this stress, or explained how it is maintained. This step, however, seems to me to be an important one, as it

{The subject of the stress in the medium. will be further considered in the Supriementary Volume, it may however be noticed here that the problem of finding a of stresses which will produce the same forces as those existing in the electric field ]B one which has an infinite nuuiber of solutions. That adopted by Maxwell is one that could not in general be produced by strains in an elastic solid.}

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122.] These diagrams should be studied as illustrations of the lanouace of Faraday in speaking of 1 lines of force,' the 1 forces of an electrified body,' &c.

The word Force denotes a restricted aspect of that action between two material bodies by which their motions are rendered different from what they would have been in the absence of that action. The whole phenomenon, when both bodies are contemplated at once, is called Stress, ind may be described as a transference of momentum from one body to the other. When we restrict our attention to the first of the two bodies, we call the stress actino, on it the Moving Force, or simply the Force on that body, and it is measured by the momentum which that body is receiving per unit of time.

The mechanical action between two charged bodies is a stress, and that on one of them is a force. The force on a small charged body is proportional to its own charge, and the force per unit of charge is called the Intensity of the force.

The word Induction was employed by Faraday to denote the mode in which the charges of electrified bodies are related to each other, every unit of positive charge being connected with a unit of negative charge by a line, the direction of which, in fluid dielectrics, coincides at every part of its course with that of the electric intensity. Such a line is often called a line of Force, but it is more correct to call it a line of Induction.

Now the quantity of electricity in a body is measured, according to Faraday's ideas, by the @@un@beg, of lines of force, or rather of induction, which proceed from it. These lines of force must all terminate somewhere, either on bodies in the neighbourhood, or on the walls and roof of the room, or on the earth, or on the heavenly bodies, and wherever they terminate there is a quantity of electricity exactly equal and opposite to that on the part of the body from which they proceeded. By examining the diagi.ams this will be seen to be the case. There is therefore no contradiction between Faraday's views and the mathematical results of the old theory, but, on the contrary, the idea of lines of force throws great light on these results, and seems to afford the means of rising by a continuous process from the somewhat rigid conceptions of the old theory to notions which may be capable of greater expansion, so as to provide room for the increase of our knowledge by further researches.



